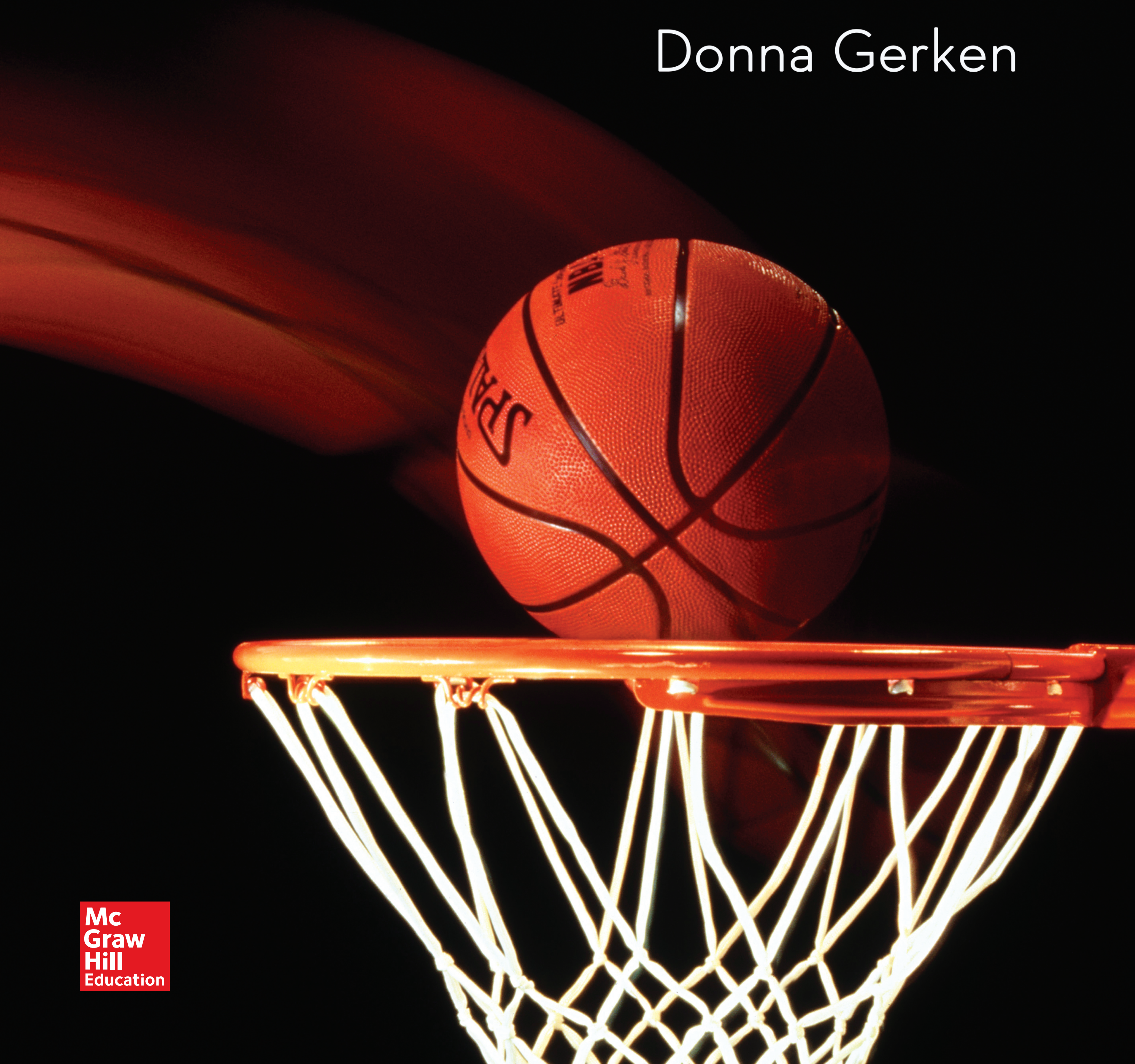
Precalculus

Julie Miller
Donna Gerken



Mc Graw Hill Education

Precalculus

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PRECALCULUS

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About the Authors

Julie Miller is from Daytona State College where she has taught developmental and upper-level mathematics courses for 20 years. Prior to her work at DSC, she worked as a software engineer for General Electric in the area of flight and radar simulation. Julie earned a bachelor of science in applied mathematics from Union College in Schenectady, New York, and a master of science in mathematics from the University of Florida. In addition to this textbook, she has authored textbooks in developmental mathematics, trigonometry, and precalculus, as well as several short works of fiction and nonfiction for young readers.

"My father is a medical researcher, and I got hooked on math and science when I was young and would visit his laboratory. I remember doing simple calculations with him and using graph paper to plot data points for his experiments. He would then tell me what the peaks and features in the graph meant in the context of his experiment. I think that applications and hands-on experience made math come alive for me, and I'd like to see math come alive for my students."

Donna Gerken is a professor at Miami Dade College where she has taught developmental courses, honors classes, and upper-level mathematics classes for decades. Throughout her career she has been actively involved with many projects at Miami Dade including those on computer learning, curriculum design, and the use of technology in the classroom. Donna's bachelor of science in mathematics and master of science in mathematics are both from the University of Miami.

Letter from the Authors

Precalculus is a foundation course essential to success in Calculus and all subsequent courses in science, technology, engineering, and mathematics. To support student success in this course, we accommodate a variety of learning styles by bringing together a seamless integration of print and digital content. The clear, concise writing style and pedagogical features of our textbook continue throughout the online content in ConnectMath, throughout our instructional videos, and throughout the adaptive reading and learning experience of SmartBook. Furthermore, to enhance preparation for Calculus, we have dedicated exercise sets that specifically target skills needed in Calculus.

The main objectives of this Precalculus textbook and its print and digital content are threefold:

- To provide students with a clear and logical presentation of fundamental concepts that will prepare them for continued study in mathematics.
- To help students develop logical thinking and problem-solving skills that will benefit them in all aspects of life.
- To motivate students by demonstrating the significance of mathematics in their lives through practical applications.

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Dedications

To our students: Enjoy your journey with mathematics and the doors it opens to science, business, and the natural world around you. —Julie Miller

For Eddie and Aurora—true friendship needs no words, but it does require the ability to appear interested while someone talks about trigonometry applications during every shared holiday dinner. —Donna Gerken

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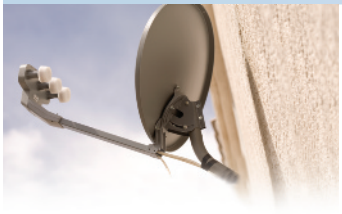

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Key Features

Clear, Precise Writing

Because a diverse group of students take this course, Julie Miller and Donna Gerken have written this manuscript to use simple and accessible language. Through their friendly and engaging writing style, students are able to understand the material easily.

Exercise Sets

The exercises at the end of each section are graded, varied, and carefully organized to maximize student learning:

- **Prerequisite Review Exercises** begin the section-level exercises and ensure that students have the foundational skills to complete the homework sets successfully.
- **Concept Connections** prompt students to review the vocabulary and key concepts presented in the section.
- **Core Exercises** are presented next and are grouped by objective. These exercises are linked to examples in the text and direct students to similar problems whose solutions have been stepped-out in detail.
- **Mixed Exercises** do *not* refer to specific examples so that students can dip into their mathematical toolkit and decide on the best technique to use.
- **Write About It** exercises are designed to emphasize mathematical language by asking students to explain important concepts.
- **Technology Connections** require the use of a graphing utility and are found at the end of exercise sets. They can be easily skipped for those who do not encourage the use of calculators.
- **Expanding Your Skills Exercises** challenge and broaden students' understanding of the material.

Problem Recognition Exercises

Problem Recognition Exercises appear in strategic locations in each chapter of the text. These exercises provide students with an opportunity to synthesize multiple concepts and decide which problem-solving technique to apply to a given problem.

Exercises for Calculus

- **Algebra for Calculus** exercises specifically target the skills students use in calculus such as the simplification of a limit, and the simplification after taking a derivative.
- **Equations and Inequalities for Calculus** is another exercise set that provide sample equations and inequalities seen in calculus. These include the important skills of finding the values that make an expression zero or undefined to find the critical numbers in calculus.

Examples

- The examples in the textbook are stepped-out in detail with thorough annotations at the right explaining each step.
- Following each example is a similar **Skill Practice** exercise to engage students by practicing what they have just learned.
- For the instructor, references to an even-numbered exercise are provided next to each example. These exercises are highlighted with blue circles in the exercise sets and mirror the related examples. With increased demands on faculty time, this has been a popular feature that helps faculty write their lectures and develop their presentation of material. If an instructor presents all of the highlighted exercises, then each objective of that section of text will be covered.

Modeling and Applications

One of the most important tools to motivate our students is to make the mathematics they learn meaningful in their lives. The textbook is filled with robust applications and numerous opportunities for mathematical modeling for those instructors looking to incorporate these features into their course.

Callouts

Throughout the text, popular tools are included to highlight important ideas. These consist of:

- **Tip** boxes that offer additional insight to a concept or procedure.
- **Avoiding Mistakes** boxes that fend off common mistakes.
- **Point of Interest** boxes that offer interesting and historical mathematical facts.
- **Instructor Notes** to assist with lecture preparation.

Graphing Calculator Coverage

Material is presented throughout the book illustrating how a graphing utility can be used to view a concept in a graphical manner. The goal of the calculator material is not to replace algebraic analysis, but rather, to enhance understanding with a visual approach. Graphing calculator examples are placed in self-contained boxes and may be skipped by instructors who choose not to implement the calculator. Similarly, the graphing calculator exercises are found at the end of the exercise sets and may also be easily skipped.

End-of-Chapter Materials

The textbook has the following end-of-chapter materials for students to review before test time:

- Brief summary with references to key concepts. A detailed summary is located at www.mhhe.com/millerprecalculus.
- Chapter review exercises.
- Chapter test.
- Cumulative review exercises. These exercises cover concepts in the current chapter as well as all preceding chapters.

Supplement Package

Supplements for the Instructor

Author-Created Digital Media

Digital assets were created exclusively by the author team to ensure that the author voice is present and consistent throughout the supplement package.

- Donna Gerken ensures that each algorithm in the online homework has a stepped-out solution that matches the textbook's writing style.
- Julie Miller created **video content** (lecture videos, exercise videos, graphing calculator videos, and Excel videos) to give students access to classroom-like instruction by the author.
- Julie Miller constructed over 50 **dynamic math animations** to accompany the text. The animations are diverse in scope and give students an interactive approach to conceptual learning. The animated content illustrates difficult concepts by leveraging the use of on-screen movement where static images in the text may fall short. They are organized in Connect hosted by ALEKS by chapter and section.

The *Instructor's Resource Manual* (IRM) is a printable electronic supplement put together by the author team. The IRM includes Guided Lecture Notes, Classroom Activities using Wolfram Alpha, and Group Activities.

- The Guided Lecture Notes are keyed to the objectives in each section of the text. The notes step through the material with a series of questions and exercises that can be used in conjunction with lecture.
- The Classroom Activities using Wolfram Alpha promote active learning in the classroom by using a powerful online resource.
- A Group Activity is available for each chapter of the book to promote classroom discussion and collaboration.

The *Instructor's Solution Manual* provides comprehensive, worked-out solutions to all exercises in the section exercises, review exercises, problem recognition exercises, chapter tests, and cumulative reviews. The steps shown in the solutions match the style and methodology of solved examples in the textbook.

TestGen is a computerized test bank utilizing algorithm-based testing software to create customized exams quickly. This user-friendly program enables instructors to search for questions by topic, format, or difficulty level; to edit existing questions, or to add new ones; and to scramble questions and answer keys for multiple versions of a single test. Hundreds of text-specific, open-ended, and multiple-choice questions are included in the question bank.

Annotated Instructor's Edition

- Answers to exercises appear adjacent to each exercise set, in a color used only for annotations.
- Instructors will find helpful notes within the margins to consider while teaching.
- References to even-numbered exercises appear in the margin next to each example for the instructor to use as Classroom Examples.

Power Points present key concepts and definitions with fully editable slides that follow the textbook. An instructor may project the slides in class or post to a website in an online course.

Supplements for the Student

Student Worksheets including guided lecture notes that step through the key objectives and Problem Recognition Exercise worksheets.

ALEKS® Prep for Precalculus
ALEKS Prep for Precalculus focuses on prerequisites and introductory material for Precalculus. These prep products can be used during the first 3 weeks of a course to prepare students for future success in the course and to increase retention and pass rates.

Connect Math® Hosted by ALEKS
Connect Math Hosted by ALEKS Corp. is an exciting, new assignment and assessment ehomework platform. Starting with an easily viewable, intuitive interface, students will be able to access key information, complete homework assignments, and utilize an integrated, media-rich eBook.

Smartbook® is the first and only adaptive reading experience available for the higher education market. Powered by the intelligent and adaptive LearnSmart engine, SmartBook facilitates the reading process by identifying what content a student knows and doesn't know. As a student reads, the material continously adapts to ensure the student is focused on the content he or she needs the most to close specific knowledge gaps.

Detailed Chapter Summaries are available at www.mhhe.com/millerprecalculus.

Mc Graw Hill Education ALEKS®

Efficient. Easy to Use. Effective. Engaging.
Gives Your Students the ALEKS Advantage

Are your students prepared for your course? Do they learn at different paces? Is there inconsistency between homework and test scores? ALEKS successfully addresses these core challenges and more.

With decades of scientific research behind its creation, ALEKS offers the most advanced adaptive learning technology that is proven to increase student success in math.

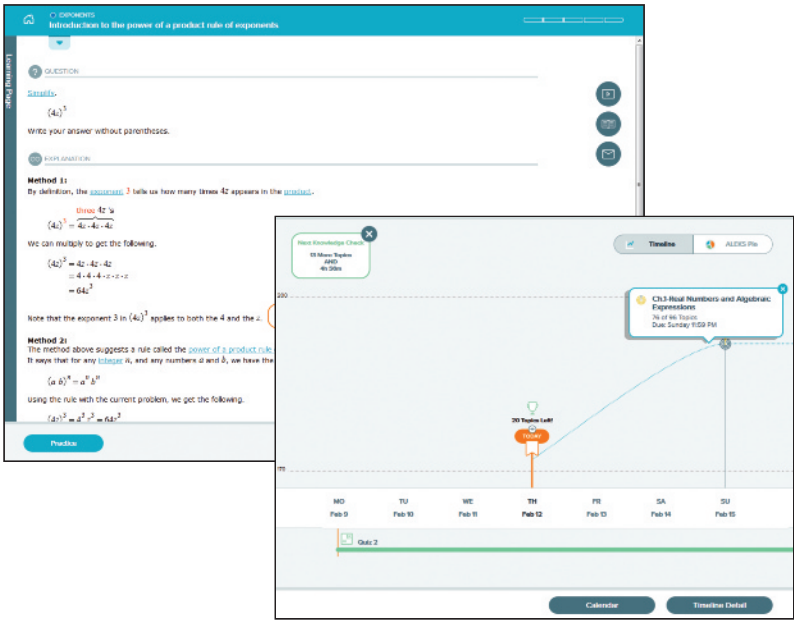
Optimized, Structured Learning

Using adaptive artificial intelligence, ALEKS identifies precisely what each student knows and doesn't know, and prescribes an individualized learning plan tailored to their unique strengths and weaknesses:

- Targets critical knowledge gaps
- Open-response environment
- Motivates student learning
- Presents only topics students are ready to learn
- Enhances learning with interactive resources
- Provides a structured learning path

Student-Friendly Learning Experience

ALEKS is designed to meet the needs of today's students. A clean, modern, mobile-ready interface allows students to easily navigate their learning, track their progress and manage their assignments from anywhere.



"I evaluated many different options, and ALEKS provided, by far, the best cycle of assessment and learning that allows for individualized instructional paths . . . no other program matches ALEKS."

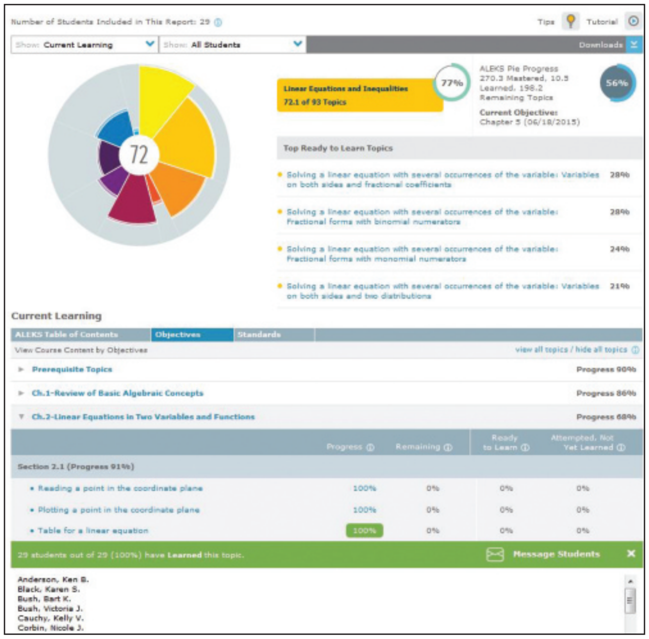
—Professor Eliza Gallagher, *Clemson University, SC*

Learn More: **Successinmath.com**

THE ALEKS Instructor Module includes intuitive customization and management features that help save you valuable time and effort. Easily manage your courses and track student progress, all through one simple interface.

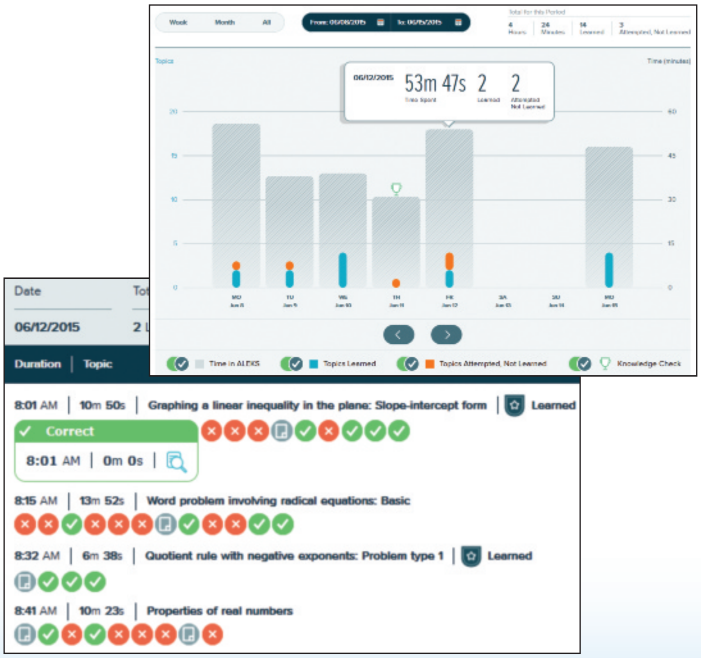
- Create learning goals with due dates that align with your textbook/syllabi and to pace student learning
- Implementation services and training help make setup simple and timely
- 100% mobile-ready allows you to manage your classes from anywhere
- LMS integration offers single sign-on and gradebook sync capabilities

ALEKS Reporting provides detailed data on student progress, allowing you to quickly identify intervention needs and to better understand student learning behaviors. View data for individual students up to the multi-campus level.



View student progress in a course area or at the individual topic level, including which topics students are struggling with the most.

Use this data to inform your teaching, group students based on similar knowledge levels, and shape a meaningful learning experience for your students.



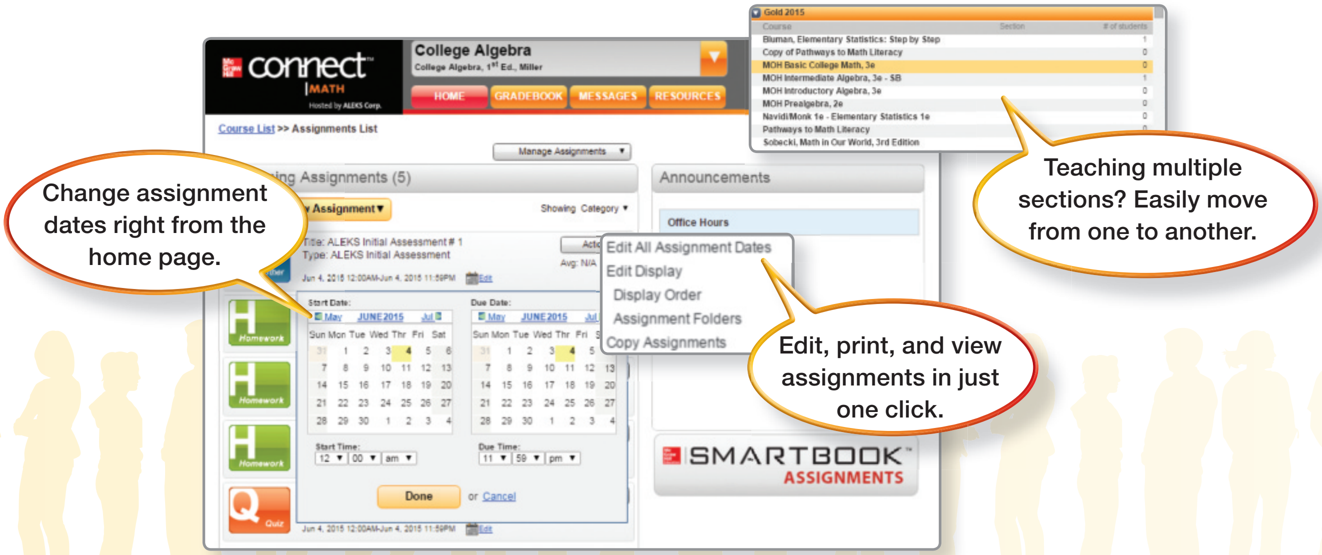
View a daily breakdown of how students learn in ALEKS, including the exact problems they attempt and their answers. This will help you to identify common mistakes and understand their behavior pattern, such as when they study and how frequently.

Learn More: **Successinmath.com**

Connect Math Hosted by ALEKS

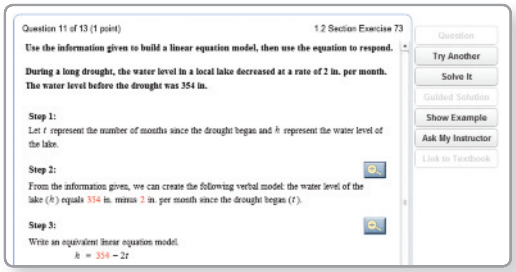
Built By Today's Educators,
For Today's Students

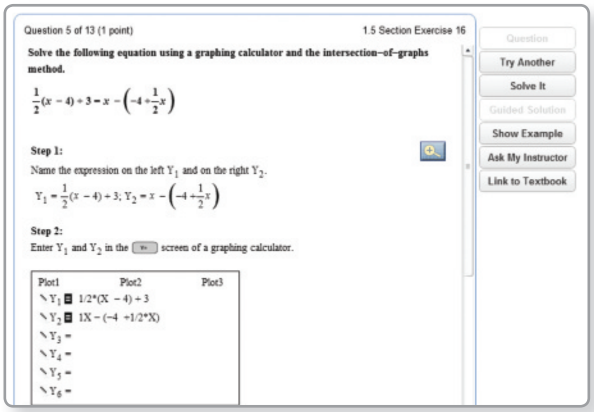
Fewer clicks means more time for you…



…and your students.

Online Exercises were carefully selected and developed to provide a seamless transition from textbook to technology.





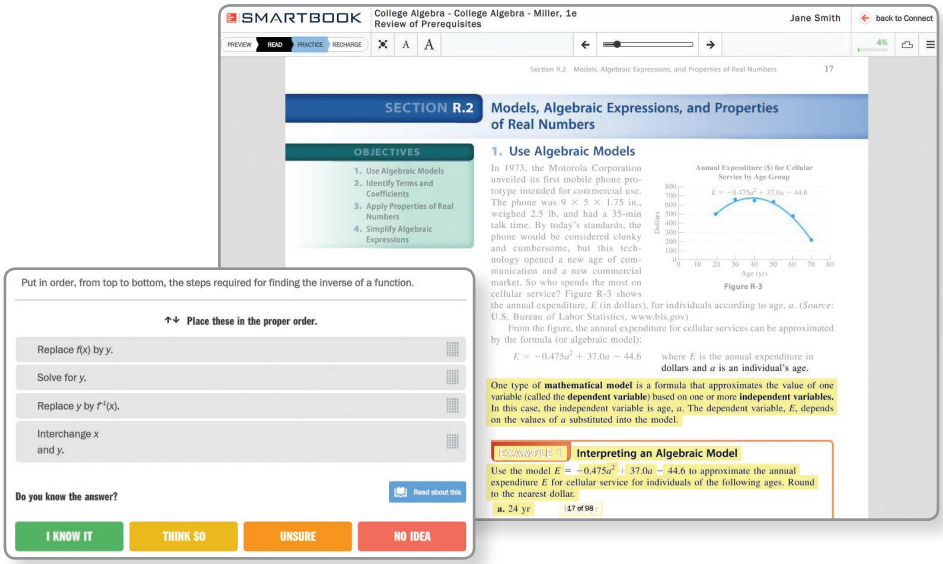
For consistency, the guided solutions match the style and voice of the original text as though the author is guiding the students through the problems.

Quality Content For Today's Online Learners

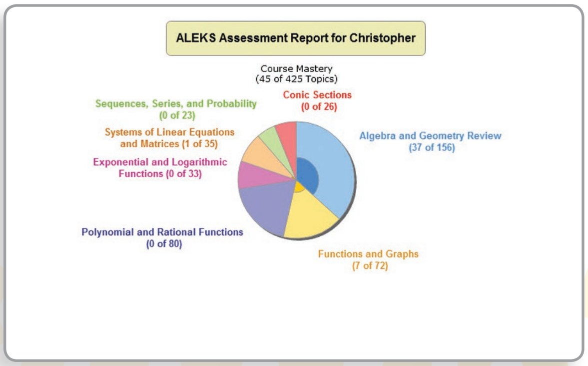
Why SmartBook? Because it's more than just words on a page.

SMARTBOOK™ Students today thrive on efficiency, mobility, and motivation. And SmartBook delivers. SmartBook is the first adaptive reading experience designed to change the way students learn. Students and instructors can enjoy access to SmartBook anywhere, anytime (now available offline) with a new and improved mobile interface. If students still prefer holding a text as they study, they can **order a loose-leaf copy of their textbook for a significant discount.**

SmartBook breaks down the learning experience into four stages: *Preview, Read, Practice,* and *Recharge*. Each stage provides personalized guidance and just-in-time remediation to ensure students stay focused and learn as efficiently as possible. With LearnSmart technology, questions are designed to foster critical thinking and conceptual learning.



Reports are also available to both students and instructors that track progress and show each student's strengths and weaknesses. What does this mean for you? Teach a more informed classroom and provide more personalized guidance.



The ALEKS® Initial Assessment is an artificially intelligent (AI), diagnostic assessment that identifies precisely what a student knows. Instructors can then use this information to make more informed decisions on what topics to cover in more detail with the class.

Our Commitment to Market Development and Accuracy

McGraw-Hill's Development Process is an ongoing, never-ending, market-oriented approach to building accurate and innovative print and digital products. We begin developing a series by partnering with authors who desire to make an impact within their discipline to help students succeed. Next, we share these ideas and manuscript with instructors for review for feedback and to ensure that the authors' ideas represent the needs within that discipline. Throughout multiple drafts, we help our authors adapt to incorporate ideas and suggestions from reviewers to ensure that the series carries the same pulse as today's classrooms. With any new series, we commit to accuracy across the series and its supplements. In addition to involving instructors as we develop our content, we also utilize accuracy checks through our various stages of development and production. The following is a summary of our commitment to market development and accuracy:

1. 3 drafts of author manuscript
2. 2 rounds of manuscript review
3. 3 rounds of accuracy checking
4. 3 rounds of proofreading and copyediting
5. Toward the final stages of production, we incorporate additional rounds of quality assurance from instructors as they help contribute toward our digital content and print supplements

This process then will start again immediately upon publication in anticipation of the next edition. With our commitment to this process, we are confident that our series has the most developed content the industry has to offer, thus pushing our desire for quality and accurate content that meets the needs of today's students and instructors.

Acknowledgments:

Paramount to the development of *Precalculus* was the invaluable feedback provided by the instructors from around the country who reviewed the manuscript or attended a market development event over the course of the several years the text was in development.

A Special Thanks to All of the Event Attendees Who Helped Shape *Precalculus*.

Focus groups and symposia were conducted with instructors from around the country to provide feedback to editors and the authors and ensure the direction of the text was meeting the needs of students and instructors.

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—Julie Miller and Donna Gerken

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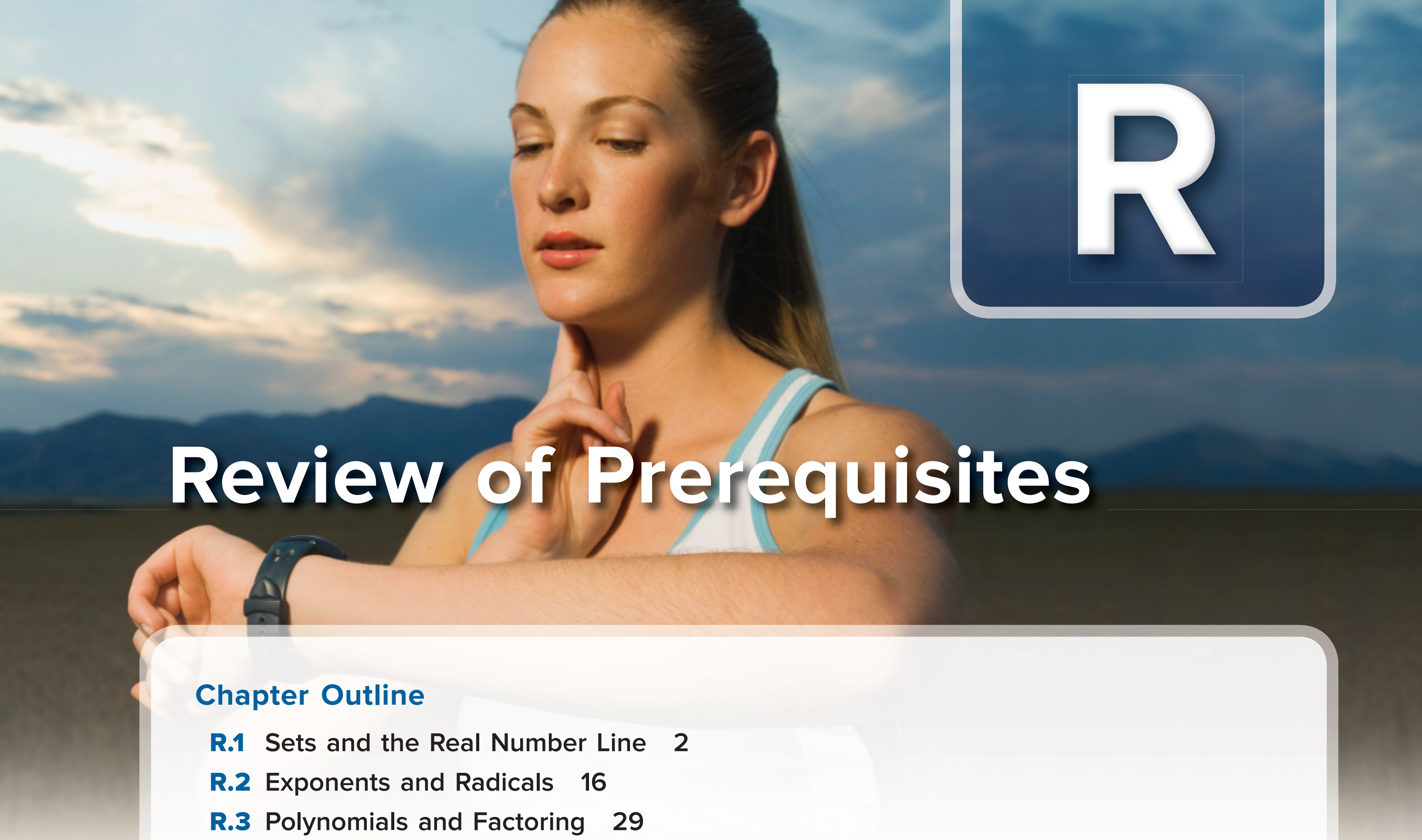


R

Review of Prerequisites

Chapter Outline

Athletes know that in order to optimize their performance they need to pace themselves and be mindful of their target heart rate. For example, a 25-year-old with a maximum heart rate of 195 beats per minute should strive for a target heart rate zone of between 98 and 166 beats per minute. This correlates to between 50% and 85% of the individual's maximum recommended heart rate (Source: American Heart Association, www.americanheart.org). The mathematics involved in finding maximum recommended heart rate and an individual's target heart rate zone use a linear model relating age and resting heart rate. An introduction to modeling is presented here in Chapter R along with the standard order of operations used to carry out these calculations.

Chapter R reviews skills and concepts required for success in college algebra. Just as an athlete must first learn the basics of a sport and build endurance and speed, a student studying mathematics must focus on necessary basic skills to prepare for the challenge ahead. Preparation for algebra is comparable to an athlete preparing for a sporting event. Putting the time and effort into the basics here in Chapter R will be your foundation for success in later chapters.

SECTION R.1 Sets and the Real Number Line

OBJECTIVES

1. Identify Subsets of the Set of Real Numbers
2. Use Inequality Symbols and Interval Notation
3. Find the Union and Intersection of Sets
4. Evaluate Absolute Value Expressions
5. Use Absolute Value to Represent Distance
6. Apply the Order of Operations
7. Simplify Algebraic Expressions
8. Write Algebraic Models

1. Identify Subsets of the Set of Real Numbers

A hybrid vehicle gets 48 mpg in city driving and 52 mpg on the highway. The formula $A = \frac{1}{48}c + \frac{1}{52}h$ gives the amount of gas A (in gal) for c miles of city driving and h miles of highway driving. In the formula, A, c, and h are called **variables** and these represent values that are subject to change. The values $\frac{1}{48}$ and $\frac{1}{52}$ are called **constants** because their values do not change in the formula.

For a trip from Houston, Texas, to Dallas, Texas, a motorist travels 36 mi of city driving and 91 mi of highway driving. The amount of fuel used by this hybrid vehicle is given by

$$A = \frac{1}{48}(36) + \frac{1}{52}(91)$$
$$= 2.5 \text{ gal}$$

The numbers used in day-to-day life such as those used to determine fuel consumption come from the set of real numbers, denoted by $\mathbb{R}$. A **set** is a collection of items called **elements.** The braces { and } are used to enclose the elements of a set. For example, {gold, silver, bronze} represents the set of medals awarded to the top three finishers in an Olympic event. A set that contains no elements is called the **empty set** (or **null set**) and is denoted by { } or $\varnothing$.

When referring to individual elements of a set, the symbol $\in$ means "is an element of," and the symbol $\notin$ means "is not an element of." For example,

$5 \in \{1, 3, 5, 7\}$ is read as "5 is an element of the set of elements 1, 3, 5, and 7."

$6 \notin \{1, 3, 5, 7\}$ is read as "6 is *not* an element of the set of elements 1, 3, 5, and 7."

A set can be defined in several ways. Listing the elements in a set within braces is called the **roster method.** Using the roster method, the set of the even numbers between 0 and 10 is represented by {2, 4, 6, 8}. Another method to define this set is by using **set-builder notation.** This uses a description of the elements of the set. For example,

$$\{x \mid x \text{ is an even number between 0 and 10}\}$$

The set of   all x such that x is an even number between 0 and 10

In our study of college algebra, we will often refer to several important **subsets** (parts of) the set of real numbers (Table R-1).

TIP Notice that the first five letters of the word *rational* spell *ratio*. This will help you remember that a rational number is a *ratio* of integers.

Table R-1 Subsets of the Set of Real Numbers, $\mathbb{R}$

Set	Definition
Natural numbers, $\mathbb{N}$	$\{1, 2, 3, \ldots\}$
Whole numbers, $\mathbb{W}$	$\{0, 1, 2, 3, \ldots\}$
Integers, $\mathbb{Z}$	$\{\cdots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
Rational numbers, $\mathbb{Q}$	$\left\{\frac{p}{q} \,\middle\vert\, p, q \in \mathbb{Z} \text{ and } q \neq 0\right\}$ • Rational numbers can be expressed as a ratio of integers where the denominator is not zero. Examples: $-\frac{6}{11}$ (ratio of -6 and 11) and 9 (ratio of 9 and 1). • All terminating and repeating decimals are rational numbers. Examples: 0.71 (ratio of 71 and 100), $0.\overline{6} = 0.666\ldots$ (ratio of 2 and 3).
Irrational numbers, $\mathbb{H}$	Irrational numbers are real numbers that cannot be expressed as a ratio of integers. The decimal form of an irrational number is nonterminating and nonrepeating. Examples: π and $\sqrt{2}$

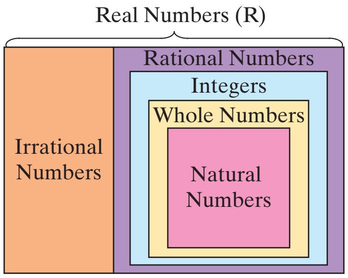


Figure R-1

The relationships among the subsets of real numbers defined in Table R-1 are shown in Figure R-1. In particular, notice that together the elements of the set of rational numbers and the set of irrational numbers make up the set of real numbers.

2. Use Inequality Symbols and Interval Notation

All real numbers can be located on the real number line. We say that a is less than b (written symbolically as $a < b$) if a lies to the left of b on the number line. This is equivalent to saying that b is greater than a (written symbolically as $b > a$) because b lies to the right of a.

$a < b$ is equivalent to $b > a$

In Table R-2, we summarize other symbols used to compare two real numbers.

Table R-2 **Summary of Inequality Symbols and Their Meanings**

Inequality	Verbal Interpretation	Other Implied Meanings	Numerical Examples
$a < b$	a is less than b	b exceeds a b is greater than a	$5 < 7$
$a > b$	a is greater than b	a exceeds b b is less than a	$-3 > -6$
$a \leq b$	a is less than or equal to b	a is at most b a is no more than b	$4 \leq 5$ $5 \leq 5$
$a \geq b$	a is greater than or equal to b	a is no less than b a is at least b	$9 \geq 8$ $9 \geq 9$
$a = b$	a is equal to b		$-4.3 = -4.3$
$a \neq b$	a is not equal to b		$-6 \neq -7$
$a \approx b$	a is approximately equal to b		$-12.99 \approx -13$

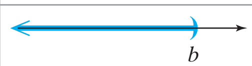
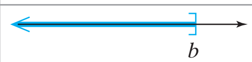
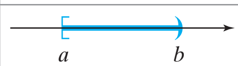
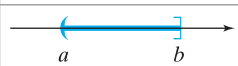
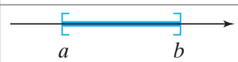
Point of Interest

The infinity symbol ∞ is called a lemniscate from the Latin *lemniscus* meaning "ribbon." English mathematician John Wallis is credited with introducing the symbol in the seventeenth century. The symbols $-\infty$ and ∞ are not themselves real numbers, but instead refer to quantities without bound or end.

An interval on the real number line can be represented in set-builder notation or in interval notation. In Table R-3, observe that a parenthesis) or (indicates that an endpoint is not included in an interval. A bracket] or [indicates that an endpoint *is* included in the interval. The real number line extends infinitely far to the left and right. We use the symbols $-\infty$ and ∞ to denote the unbounded behavior to the left and right, respectively.

Table R-3 **Summary of Interval Notation and Set-Builder Notation**

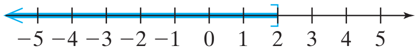
Let a, b, and x represent real numbers.

Set-Builder Notation	Verbal Interpretation	Graph	Interval Notation
$\{x \mid x > a\}$	the set of real numbers greater than a		(a, ∞)
$\{x \mid x \geq a\}$	the set of real numbers greater than or equal to a		$[a, \infty)$
$\{x \mid x < b\}$	the set of real numbers less than b		$(-\infty, b)$
$\{x \mid x \leq b\}$	the set of real numbers less than or equal to b		$(-\infty, b]$
$\{x \mid a < x < b\}$	the set of real numbers between a and b		(a, b)
$\{x \mid a \leq x < b\}$	the set of real numbers greater than or equal to a and less than b		$[a, b)$
$\{x \mid a < x \leq b\}$	the set of real numbers greater than a and less than or equal to b		$(a, b]$
$\{x \mid a \leq x \leq b\}$	the set of real numbers between a and b, inclusive		$[a, b]$
$\{x \mid x \text{ is a real number}\}$ $\mathbb{R}$	the set of all real numbers		$(-\infty, \infty)$

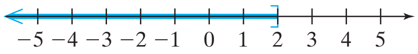
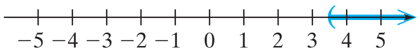
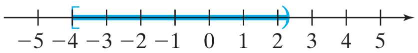
TIP As an alternative to using parentheses and brackets to represent the endpoints of an interval, an open dot or closed dot may be used. For example, $\{x \mid a \leq x < b\}$ would be represented as follows.

EXAMPLE 1 Expressing Sets in Interval Notation and Set-Builder Notation

Complete the table.

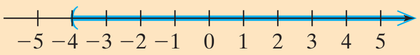
Graph	Interval Notation	Set-Builder Notation
		
	$\left(\frac{7}{2}, \infty\right)$	
		$\{y \mid -4 \leq y < 2.3\}$

Solution:

Graph	Interval Notation	Set-Builder Notation	Comments
	$(-\infty, 2]$	$\{x \mid x \leq 2\}$	The bracket at 2 indicates that 2 is included in the set.
	$\left(\frac{7}{2}, \infty\right)$	$\{x \mid x > \frac{7}{2}\}$	The parenthesis at $\frac{7}{2} = 3.5$ indicates that $\frac{7}{2}$ is *not* included in the set.
	$[-4, 2.3)$	$\{y \mid -4 \leq y < 2.3\}$	The set includes the real numbers between -4 and 2.3, including the endpoint -4.

Skill Practice 1

a. Write the set represented by the graph in interval notation and set-builder notation.



b. Given the interval, $\left(-\infty, -\frac{4}{3}\right]$, graph the set and write the set-builder notation.

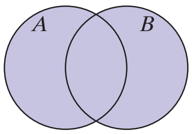
c. Given the set, $\{x \mid 1.6 < x \leq 5\}$, graph the set and write the interval notation.

3. Find the Union and Intersection of Sets

Two or more sets can be combined by the operations of union and intersection.

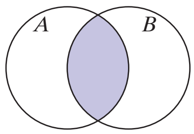
Union and Intersection of Sets

The **union** of sets A and B, denoted $A \cup B$, is the set of elements that belong to set A or to set B or to both sets A and B.



$A \cup B$
A union B
The elements in A or B or both

The **intersection** of sets A and B, denoted $A \cap B$, is the set of elements common to both set A and set B.



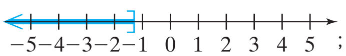
$A \cap B$
A intersection B
The elements common to A and B

In Examples 2 and 3, we practice finding the union and intersections of sets.

Answers

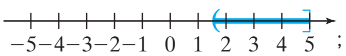
1. a. Interval notation: $(-4, \infty)$;
Set-builder notation: $\{x \mid x > -4\}$

b.



Set-builder notation: $\{x \mid x \leq -\frac{4}{3}\}$

c.



Interval notation: $(1.6, 5]$

EXAMPLE 2 Finding the Union and Intersection of Sets

Find the union or intersection of sets as indicated.

$A = \{-5, -3, -1, 1\}$ $B = \{-5, 0, 5\}$ $C = \{-4, -2, 0, 2, 4\}$

a. $A \cap B$ **b.** $A \cup B$ **c.** $A \cap C$

Solution:

a. $A \cap B = \{-5\}$ — The only element common to both A and B is -5. $A = \{-5, -3, -1, 1\}$, $B = \{-5, 0, 5\}$

b. $A \cup B = \{-5, -3, -1, 0, 1, 5\}$ — The union of A and B consists of all elements from A along with all elements from B.

c. $A \cap C = \{\ \}$ — Sets A and C have no common elements.

Skill Practice 2 From the sets A, B, and C defined in Example 2, find

a. $B \cap C$ **b.** $B \cup C$ **c.** $A \cup C$

EXAMPLE 3 Finding the Union and Intersection of Sets

Find the union or intersection as indicated.

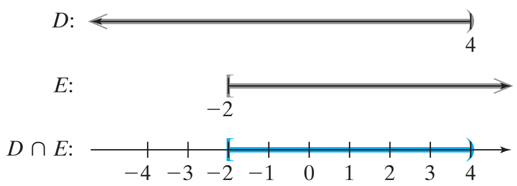
$D = \{x \mid x < 4\}$ $E = \{x \mid x \geq -2\}$ $F = \{x \mid x \leq -3\}$

a. $D \cap E$ **b.** $D \cup E$ **c.** $D \cup F$ **d.** $E \cap F$

Solution:

Graph each individual set and take the union or intersection.

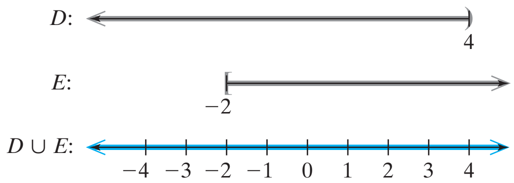
a.



The intersection is the region of overlap.

Set notation: $D \cap E = \{x \mid -2 \leq x < 4\}$
Interval notation: $[-2, 4)$

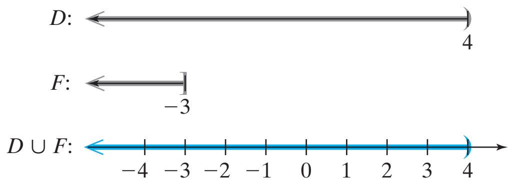
b.



The union contains the elements from D along with those from E.

Set notation: $D \cup E = \mathbb{R}$
Interval notation: $(-\infty, \infty)$

c.



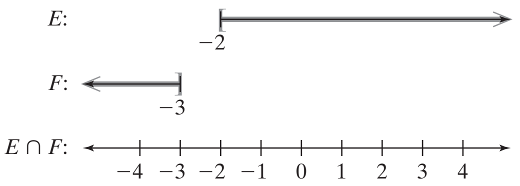
Set F is contained within set D. The union is set D itself.

Set notation: $D \cup F = \{x \mid x < 4\}$
Interval notation: $(-\infty, 4)$

Answers

2. a. $\{0\}$ **b.** $\{-5, -4, -2, 0, 2, 4, 5\}$
c. $\{-5, -4, -3, -2, -1, 0, 1, 2, 4\}$

d.



There are no elements common to both sets E and F.

Set notation: $E \cap F = \{\ \}$

Skill Practice 3 Given $X = \{x|x \geq -1\}$, $Y = \{x|x < 2\}$, and $Z = \{x|x < -4\}$, find the union or intersection of sets as indicated.

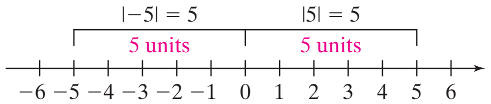
a. $X \cap Y$ **b.** $X \cup Y$ **c.** $Y \cap Z$ **d.** $X \cap Z$

4. Evaluate Absolute Value Expressions

Every real number x has an opposite denoted by $-x$. For example, $-(4)$ is the opposite of 4 and simplifies to -4. Likewise, $-(-2.1)$ is the opposite of -2.1 and simplifies to 2.1.

The **absolute value** of a real number x, denoted by $|x|$, is the distance between x and zero on the number line. For example:

$|-5| = 5$ because -5 is 5 units from zero on the number line.
$|5| = 5$ because 5 is 5 units from zero on the number line.



Notice that if a number is negative, its absolute value is the opposite of the number. If a number is positive, its absolute value is the number itself.

Definition of Absolute Value

Let x be a real number. Then $|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$

Verbal Interpretation	Numerical Example
• If x is positive or zero, then $|x|$ is just x itself.	$|4| = 4$ $|0| = 0$
• If x is negative, then $|x|$ is the opposite of x.	$|-4| = -(-4) = 4$

EXAMPLE 4 Removing Absolute Value Symbols

Use the definition of absolute value to rewrite each expression without absolute value bars.

a. $|\sqrt{3} - 3|$ **b.** $|3 - \sqrt{3}|$ **c.** $\dfrac{|x - 4|}{x - 4}$ for $x < 4$

Answers

3. a. $\{x\,|\,-1 \leq x < 2\}$; $[-1, 2)$
b. $\mathbb{R}$; $(-\infty, \infty)$
c. $\{x\,|\,x < -4\}$; $(-\infty, -4)$
d. $\{\ \}$

TIP Calculator approximations can be used to show that $\sqrt{3} - 3 \approx -1.27$ is negative, and $3 - \sqrt{3} \approx 1.27$ is positive.

Solution:

a. $|\sqrt{3} - 3| = -(\sqrt{3} - 3)$
$= -\sqrt{3} + 3 \quad \text{or} \quad 3 - \sqrt{3}$

The value $\sqrt{3} \approx 1.73 < 3$, which implies that $\sqrt{3} - 3 < 0$. Since the expression inside the absolute value bars is negative, take the opposite.

b. $|3 - \sqrt{3}| = 3 - \sqrt{3}$

The value $\sqrt{3} \approx 1.73 < 3$, which implies that $3 - \sqrt{3} > 0$. Since the expression inside the absolute value bars is positive, the simplified form is the expression itself.

c. $\dfrac{|x - 4|}{x - 4}$ for $x < 4$

$$= \frac{-(x - 4)}{x - 4}$$
$$= -1 \cdot \frac{x - 4}{x - 4}$$
$$= -1$$

The condition $x < 4$, implies that $x - 4 < 0$. Since the expression inside the absolute value bars is negative, take the opposite.

Skill Practice 4 Use the definition of absolute value to rewrite each expression without absolute value bars.

a. $|5 - \sqrt{7}|$ **b.** $|\sqrt{7} - 5|$ **c.** $\dfrac{x + 6}{|x + 6|}$ for $x > -6$

5. Use Absolute Value to Represent Distance

Absolute value is also used to denote distance between two points on a number line.

Distance Between Two Points on a Number Line

The distance between two points a and b on a number line is given by

$$|a - b| \quad \text{or} \quad |b - a|$$

That is, the distance between two points on a number line is the absolute value of their difference.

EXAMPLE 5 Determining the Distance Between Two Points

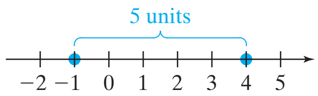
Write an absolute value expression that represents the distance between 4 and -1 on the number line. Then simplify.

Solution:

$|4 - (-1)| = |5| = 5$

$|-1 - 4| = |-5| = 5$

The distance between 4 and -1 is represented by $|4 - (-1)|$ or by $|-1 - 4|$.



Skill Practice 5 Write an absolute value expression that represents the distance between -9 and 2 on the number line. Then simplify.

Answers

4. a. $5 - \sqrt{7}$
b. $5 - \sqrt{7}$
c. 1

5. $|-9 - 2|$ or $|2 - (-9)|$; The distance is 11 units.

6. Apply the Order of Operations

TIP Note that the square of any real number is nonnegative. Therefore, there is no real number that is the square root of a negative number. For example, $\sqrt{-25}$ is not a real number because no real number when squared equals -25. *Note*: The value $\sqrt{-25}$ is an imaginary number and will be discussed in Section R.3.

Repeated multiplication can be written by using exponential notation. For example, the product $5 \cdot 5 \cdot 5$ can be written as 5^3. In this case, 5 is called the **base** of the expression and 3 is the **exponent (or power).** The exponent indicates how many times the base is used as a factor.

To find a square root of a nonnegative real number, we reverse the process to square a number. For example, a square root of 25 is a number that when squared equals 25. Both 5 and -5 are square roots of 25, because $5^2 = 25$ and $(-5)^2 = 25$. A radical sign $\sqrt{}$ is used to denote the principal square root of a number. The **principal square root** of a nonnegative real number is the square root that is greater than or equal to zero. Therefore, the principal square root of 25, denoted by $\sqrt{25}$, equals 5.

$$\sqrt{25} = 5 \text{ because } 5 \geq 0 \text{ and } 5^2 = 25$$

The symbol $\sqrt[3]{}$ represents the cube root of a number. For example:

$$\sqrt[3]{64} = 4 \text{ because } 4^3 = 64$$

Many expressions involve multiple operations. In such a case, it is important to follow the order of operations.

Order of Operations

Step 1 Simplify expressions within parentheses and other grouping symbols. These include absolute value bars, fraction bars, and radicals. If nested grouping symbols are present, start with the innermost symbols.

Step 2 Evaluate expressions involving exponents.

Step 3 Perform multiplication or division in the order in which they occur from left to right.

Step 4 Perform addition or subtraction in the order in which they occur from left to right.

EXAMPLE 6 Simplifying a Numerical Expression

Simplify. $7 - \{8 + 4[2 - (5 - \sqrt{64})^2]\}$

Solution:

$7 - \{8 + 4[2 - (5 - \sqrt{64})^2]\}$

$= 7 - \{8 + 4[2 - (5 - 8)^2]\}$ Simplify within inner parentheses, $\sqrt{64} = 8$.

$= 7 - \{8 + 4[2 - (-3)^2]\}$ Subtract within the inner parentheses.

$= 7 - [8 + 4(2 - 9)]$ Continue simplifying within the inner parentheses. Simplify $(-3)^2$ to get 9.

$= 7 - [8 + 4(-7)]$ Simplify $(2 - 9)$ to get -7.

$= 7 - (8 - 28)$ Multiply before adding or subtracting.

$= 7 - (-20)$ Simplify within parentheses.

$= 27$

Skill Practice 6 Simplify. $50 - \{2 - [\sqrt{121} + 3(-1 - 3)^2]\}$

Answer
6. 107

When simplifying an expression, particular care must be taken with expressions involving division and zero.

Division Involving Zero

To investigate division involving zero, consider the expressions $\frac{5}{0}$, $\frac{0}{5}$, and $\frac{0}{0}$ and their related multiplicative forms.

1. Division by zero is undefined.
   Example: $\frac{5}{0} = n$ implies that $n \cdot 0 = 5$. No number, n, satisfies this requirement.
2. Zero divided by any nonzero number is zero.
   Example: $\frac{0}{5} = 0$ implies that $0 \cdot 5 = 0$ which is a true statement.
3. We say that $\frac{0}{0}$ is **indeterminate** (cannot be uniquely determined). This concept is investigated in detail in a first course in calculus.
   Example: $\frac{0}{0} = n$ implies that $n \cdot 0 = 0$. This is true for any number n. Therefore, the quotient cannot be uniquely determined.

7. Simplify Algebraic Expressions

An algebraic **term** is a product of factors that may include constants and variables. An algebraic **expression** is a single term or the sum of two or more terms. For example, the expression

$$-3xz^2 + \left(-\frac{4}{b}\right) + z\sqrt{x - y} + 5 \text{ has four terms.}$$

The terms $-3xz^2$, $-\frac{4}{b}$, and $z\sqrt{x - y}$ are **variable terms.** The term 5 is not subject to change and is called a **constant term.** The constant factor within each term is called the **coefficient** of the term. Writing the expression as $-3xz^2 + \left(-4 \cdot \frac{1}{b}\right) + 1z\sqrt{x - y} + 5$, we identify the coefficients of the four terms as -3, -4, 1, and 5, respectively.

The properties of real numbers summarized in Table R-4 are often helpful when working with algebraic expressions.

Table R-4 Properties of Real Numbers

Let a, b, and c represent real numbers or real-valued expressions.

Property	In Symbols and Words	Examples
Commutative property of addition	$a + b = b + a$ The order in which real numbers are added does not affect the sum.	ex: $4 + (-7) = -7 + 4$ ex: $6 + w = w + 6$
Commutative property of multiplication	$a \cdot b = b \cdot a$ The order in which real numbers are multiplied does not affect the product.	ex: $5 \cdot (-4) = -4 \cdot 5$ ex: $x \cdot 12 = 12x$
Associative property of addition	$(a + b) + c = a + (b + c)$ The order in which real numbers are grouped under addition does not affect the sum.	ex: $(3 + 5) + 2 = 3 + (5 + 2)$ ex: $-9 + (2 + t) = (-9 + 2) + t$ $= -7 + t$
Associative property of multiplication	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$ The order in which real numbers are grouped under multiplication does not affect the product.	ex: $(6 \cdot 7) \cdot 3 = 6 \cdot (7 \cdot 3)$ ex: $8 \cdot \left(\frac{1}{8} \cdot y\right) = \left(8 \cdot \frac{1}{8}\right) \cdot y$ $= 1y$
Identity property of addition	$a + 0 = a$ and $0 + a = a$ The number 0 is called the **identity element of addition** because any number plus 0 is the number itself.	ex: $-5 + 0 = -5$ ex: $0 + \sqrt{z} = \sqrt{z}$
Identity property of multiplication	$a \cdot 1 = a$ and $1 \cdot a = a$ The number 1 is called the **identity element of multiplication** because any number times 1 is the number itself.	ex: $\sqrt{2} \cdot 1 = \sqrt{2}$ ex: $1 \cdot (2w + 3) = 2w + 3$

(Continued)